Dedekind Cuts, Real, and Rational Numbers

I believe the statement, “between any two distinct real numbers, there is a rational number” is true. The statement is true for the same reason why Zeno thought his midway paradox to be true. Zeno’s midway paradox being that in order for an object to move it would have to travel to an infinite number of midpoint. In between any two distinct rational numbers lie numbers between the two. So, there would be a rational number between any two real numbers. In fact there would be infinitely many rational numbers in between any two real numbers. It’s the same as there being a rational number in between any set of rational numbers. The rational numbers are continuous because there can be infinitely many rational numbers in between a pair of any two rational numbers. Another reason why the rational numbers are continuous is that they can be infinitely large. On the other hand if you were to take the square root of every number on the number line then there would not exist an infinite number of rational numbers between any two of the numbers. Because of this the square roots of rational numbers are not continuous. The rational numbers on the number line are continuous.